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Hiroshi Michiwaki

The law of conservation of energy and the law of conservation of momentum in the special theory of relativity and division by zero

The formula for the relativistic mass is given by

$$m(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (1)$$

This can be represented in a more general manner as follows by multiplying both sides by c^2.

$$E = m(v)c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (2)$$

This formula gives the total energy in a system.

Consider a situation in which an object with a rest mass of m_0 (for example, a neutrino) is traveling at speed c. Then, according to division by zero,

$$E = m(c)c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{c^2}{c^2}}} = \frac{m_0 c^2}{0} = 0 \qquad (3)$$

This would imply that the total energy in the system is 0, which contradicts reality as seen through experiments. Incidentally, in the case of reducible set theoretical division by zero, Equation (3) becomes

$$E = m(c)c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_0 c^2}{0} = 0 \cdots m_0 c^2 \qquad (4)$$

This implies that the energy in the system has been conserved.

Similarly, multiplying both sides of Equation (1) by the speed of travel v (normally, momentum is a vector quantity, but to make it easier to understand the essence of this calculation, we treat it as a scalar here) gives

$$P = m(v)v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (5)$$

Similar to Equation (3), consider a situation in which an object with a rest mass of m_0 (for example, a neutrino) is traveling at speed c. Then, according to division by zero,

$$P = m(c)c = \frac{m_0 c}{\sqrt{1 - \frac{c^2}{c^2}}} = \frac{m_0 c}{0} = 0 \qquad (6)$$

This result also contradicts the law of conservation of momentum. Of course, according to reducible set theoretical division by zero, Equation (6) becomes

$$P = m(c)c = \frac{m_0 c}{\sqrt{1 - \frac{c^2}{c^2}}} = \frac{m_0 c}{0} = 0 \cdots m_0 c \qquad (7)$$

This implies that the momentum in the system has been conserved.