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The Pythagorean theorem on a circle and division by zero

Let θ refer to the included angle that is formed between the x-axis and the line segment OP that connects a point P on the unit circle and the origin O. Then, the coordinates of point P are ($\cos\theta$, $\sin\theta$). Therefore, we obtain

$$\cos\theta = \frac{1-\sin^2\theta}{\cos\theta} \qquad (1)$$

and

$$\sin\theta = \frac{1-\cos^2\theta}{\sin\theta} \qquad (2)$$

Here, Equation (1) and Equation (2) apply to all values of θ.

For example, if we let $\theta = \pi/2$ in Equation (1), then we obtain

$$\cos\frac{\pi}{2} = \frac{1-\sin^2\frac{\pi}{2}}{\cos\frac{\pi}{2}} = \frac{1-1^2}{0} = \frac{0}{0} = 0 \qquad (3)$$

Furthermore, if we let $\theta = 0$ in Equation (2), then we obtain

$$\sin 0 = \frac{1-\cos^2 0}{\sin 0} = \frac{1-1^2}{0} = \frac{0}{0} = 0 \qquad (4)$$

Equation (3) and Equation (4) follow from the fundamental theorem of division by zero.
□

When reducing two functions, it is necessary to use caution in cases in which the value of the function is 0. Of course, considering a point P at a radius r, we obtain a similar result whether we consider

$$x = \frac{r^2-(r\sin\theta)^2}{r\cos\theta} \qquad (5)$$

or

$$y = \frac{r^2-(r\cos\theta)^2}{r\sin\theta} \qquad (6)$$

When the radius satisfies $r = 0$, Equation (5) and Equation (6) become

$$x = \frac{r^2 - (r\sin\theta)^2}{r\cos\theta} = \frac{0^2 - (0\cdot\sin\theta)^2}{0\cdot\cos\theta} = \frac{0}{0} = 0 \qquad (7)$$

and

$$y = \frac{r^2 - (r\cos\theta)^2}{r\sin\theta} = \frac{0^2 - (0\cdot\cos\theta)^2}{0\cdot\sin\theta} = \frac{0}{0} = 0 \qquad (8)$$

respectively. In this example, even if we reduce r beforehand, the result will have the same value as in Equation (7) and Equation (8). Furthermore, because

$$\frac{x}{r} = \cos\theta\,,\ \ \frac{y}{r} = \sin\theta$$

when $r = 0$, it is necessary to be aware that $\cos\theta = 0$ and $\sin\theta = 0$. In other words, when this fact is considered, Equation (7) and Equation (8) become

$$x = \frac{r^2 - (r\sin\theta)^2}{r\cos\theta} = \frac{0^2 - (0\cdot 0)^2}{0\cdot 0} = \frac{0}{0} = 0 \qquad (9)$$

and

$$y = \frac{r^2 - (r\cos\theta)^2}{r\sin\theta} = \frac{0^2 - (0\cdot 0)^2}{0\cdot 0} = \frac{0}{0} = 0 \qquad (10)$$

respectively. Even when the same symbol is reduced, for example when r is reduced to $r^2/r = r$ in this case, r still remains in the numerator. Even if we set $r = 0$, the result will be the same both before and after reduction. However, because $0/0 \neq 1$ and $0/0 = 0$, it is necessary to use caution when performing the original reduction, which comprises $a/a = 1$.