

### Expanded Fractions and Division by Zero

**Theorem** Given  $a, b, c \in \mathbb{R}$ , if

$$F(a, b) - F(a - bc, b) = c, \quad F(a, b) = c + F(a - bc, b)$$

and

$$F(a, b) = \frac{a}{b} \quad (a, b \in \mathbb{R})$$

then, the following is true

$$F(a, 0) = 0$$

**Proof** In the function  $F$ , if  $b = 0$ , then

$$F(a, 0) - F(a - 0 \times c, 0) = F(a, 0) - F(a, 0) = c$$

Here,  $c$  is the leading number of the compound number  $F$ , and the quotient of  $c$  and  $(a-bc)^2$  is the quotient of  $F$ ; however, in  $F(a, 0)$ ,  $c = 0$  is the only value that can be obtained for  $c$ . Therefore,  $F(a, 0) = 0$ . QED