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Fixed-Point Theorem, Curvature, and Division by Zero

Theorem The following relational expression is true:

$$\frac{0}{0} = \frac{1}{0} = 0$$

Furthermore, when the radius of curvature r is used to let the curvature μ be $\mu(r) = 1/r$,

$$\mu(0) = 0$$

Proof Given a disk on a suitable closed interval of a polar coordinate system centered at the origin, where $0 \leqq \theta \leqq 2\pi$, the angular velocity ω, using the factor of proportionality μ, is expressed as

$$\omega = f(v) = \mu v \qquad (1)$$

Let $\mu \neq 1$ and v be a quantity called peripheral speed.

According to L.E.J. Brouwer's fixed-point theorem, there is at least one value of v_c such that for (1),

$$f(v_c) = v_c \qquad (2)$$

From this assumption, the value of v_c that satisfies (2) should clearly be $v_c = 0$, namely

$$\omega = f(0) = 0 \qquad (3)$$

Incidentally, the angular velocity ω is expressed as a time differential of the phase θ, that is,

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \qquad (4)$$

and the peripheral speed v is expressed using the arc length s as

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \qquad (5)$$

Moreover, if (1) is transformed as follows:

$$\mu = \frac{\omega}{v} \qquad (6)$$

and (4) and (5) are substituted into expression (6),

$$\mu = \frac{\omega}{v} = \frac{\mathrm{d}\theta/\mathrm{d}t}{\mathrm{d}s/\mathrm{d}t} = \frac{\mathrm{d}\theta}{\mathrm{d}s} \qquad (7)$$

it can be seen that μ represents the curvature. Consequently, the curvature (r), where r is the radius of curvature, is expressed as

$$\mu = \frac{\omega}{v} = \frac{1}{r} \qquad (8)$$

Based on this, (1) can be rewritten as

$$\omega = f(v) = \frac{v}{r} \qquad (9)$$

Moreover, (9) can be expressed as

$$v = g(r) = r\omega \qquad (10)$$

Thus, when $r = 0$ for all angular velocities ω, we have

$$v = g(0) = 0 \qquad (11)$$

Consequently, the origin of this rotating disk system is a fixed point according to L.E.J. Brouwer's fixed-point theorem.

Therefore, based on (3) and (11), the origin of a rotating disk system (r, ω, v) is a fixed point with values (0,0,0), which yields

$$\omega = f(0) = \frac{0}{0} = 0 \qquad (12)$$

in (9). Applying (12) to (8) yields

$$\mu = \frac{0}{0} = \frac{1}{0} = 0 \qquad (13)$$

In addition to showing that 0/0 = 1/0 = 0, that is, the quotient of any number by 0 is 0, (13) also shows that when the radius of curvature $r = 0$, the curvature (r) is $\mu(0) = 0$.

QED