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Super-composite Numbers as a Class of Nonnegative Integers and Division by Zero

Definition When the number of nonnegative integers that can evenly divide a given nonnegative integer is 1, that nonnegative integer is called an identity element; when it is 2, a prime number; and when it is 3 or more, a composite number. In particular, when it is 3 or more and the nonnegative integer cannot be divided by 2, it is called an odd composite number; and if it can be divided by 2, it is called an even composite number. Furthermore, a nonnegative integer that can be evenly divided by all nonnegative integers is called a super-composite number (or super-even composite number). Then, the following theorem is true.

Theorem 0 is a super-composite number.

Proof When 0 is divided by 0, according to the fundamental theorem of reducible set-theoretical division by zero

$$\frac{a}{0} = 0 \cdots a \qquad (1)$$

(where $\cdots a$ denotes the remainder of a), if $a = 0$ in (1), then $\cdots a=0$; thus, 0 is clearly evenly divisible by 0.

If n is any natural number, it is self-evident that

$$\frac{0}{n} = 0 \cdots 0 \qquad (2)$$

Based on the above, 0 is evenly divisible by all nonnegative integers. Consequently, 0 is a super-composite number according to Definition. QED

Equation (1) implies that dividing 1 by 0 yields a remainder of 1. Thus, 1 is not divisible by 0, only by 1, and is an identity element according to Definition. Furthermore, dividing a positive prime integer by 0 yields the integer as a remainder; being indivisible by only two numbers, 1 and itself, it is also a prime number according to the definition. Numbers other than 0, 1, and prime numbers yield the same number when divided by 0 but are evenly divisible by 1 as well as 2 or more different prime numbers. This implies that they are divisible by 3 or more nonnegative integers and are thus composite numbers.

Note Let it be assumed that all nonnegative integers are expressed as the product of the highest power of the different prime factors by which they are divisible. Considering the range of natural numbers, if N is any natural number, p_j the j-th prime in ascending order, and n_j the largest nonnegative integer satisfying $n_j \geq 0$ that is the highest power of the prime p_j by which 0 is evenly divisible, a natural number N satisfies

$$N = \prod_{j=1}^{\infty} p_j^{n_j}$$

as well as

$$0 \leq \sum_{j=1}^{\infty} n_j < M$$

provided that M is a sufficiently large constant.

It is now assumed that N is extended from natural numbers to nonnegative integers, that is, the zero element is added to the set N.

Based on the above theorem, 0 is a super-composite number that has all prime numbers p_j as factors; thus, n_j, i.e., the highest power of prime factors p_j, satisfies $n_j > 0$. That is,

$$\sum_{j=1}^{\dot{\infty}} n_j = \dot{\infty}$$

is true. Here, $\dot{\infty}$ is a point beyond infinity, or true infinity, and not the limit of the increase sequence. Likewise, p_j is not an extreme quantity called “infinity,” but rather a truly infinite quantity. As super-composite numbers are also divisible by the composite number $p_j^{n_j}$, which consists of the power of each prime number p_j, n_j, i.e., the highest power of prime factors p_j, has a truly infinite size.

Consequently, the super-composite number 0 can be expressed as

$$0 = \left[\!\left[\prod_{j=1}^{\dot{\infty}} p_j^{n_j} \right]\!\right]_{n_j=\dot{\infty}} = \prod_{m=1}^{\dot{\infty}} m = \dot{\infty}$$

That is, $\dot{\infty}=0$ is considered to be true.

This would also seem to make it obvious, in a sense, that nonnegative integers except 0 are not divisible by 0. The reason for this is that although 0 has all nonnegative integers as factors (i.e., it has an infinite number of divisors), all nonnegative integers except 0 (i.e., natural numbers) have only a finite number of divisors. Thus, there is no reason they should be divisible by 0, which has an infinite number of divisors.