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Basis for distance and angle, and division by zero

Theorem 1: When using radius set R and arc length set A, and angle set Θ is defined by division A/R,

$$\Theta = \frac{A}{R} = \frac{0}{0} = 0$$

holds.

Theorem 2: The center of a rotating body does not rotate.

Proof: All sets X include the empty set $\emptyset$, in other words,

$$X \supseteq \emptyset \quad (1)$$

Therefore, radius set R is

$$R = \{r\} \cup \emptyset R \quad (2)$$

if element r of R is used as $\{r \in R | 0 < r\}$. However, $\exists \emptyset R \forall \{r \notin \emptyset_R | 0 < r\}$. Similarly, the arc length set A is

$$A = \{a\} \cup \emptyset_A \quad (3)$$

if element a of A is used as $\{a \in A | 0 < a \leq 2\pi r\}$. Here, $\exists \emptyset_A \forall a \{a \notin \emptyset_A | 0 < a \leq 2\pi r\}$, and π is the ratio of the circumference of a circle to its diameter. Here, if the radius set R is fixed, considering the mapping f that maps the arc length set A to angle set Θ,

$$f : A \rightarrow \Theta \quad (4)$$

it clearly makes a bijection. This is because the arc length set A satisfies Equation (3). On the other hand, if the element θ of the angle set Θ is $\{\theta \in \Theta | 0 < \theta \leq 2\pi\}$, from Equation (1),

$$\Theta = \{\theta\} \cup \emptyset_\Theta \quad (5)$$

Here, $\exists \emptyset_\Theta \forall \theta \{\theta \notin \emptyset_\Theta | 0 < \theta \leq 2\pi\}$. At this time,

$$f : a \rightarrow \theta \quad (6)$$

is clearly a bijection. On the other hand, if the arc length set A is an empty set $\emptyset_A$, then the angle set Θ will clearly also be an empty set $\emptyset_\Theta$, and the opposite also holds, as bijection

$$f : \emptyset_A \rightarrow \emptyset_\Theta \quad (7)$$

is established. Therefore,

$$f\colon \{a\} \cup \emptyset_A \to \{\theta\} \cup \emptyset_\Theta$$
$$\therefore f\colon A \to \Theta \quad (8)$$

makes a bijection.

On the other hand, if the arc length set A is fixed, if we consider mapping g that maps the radius set R to the angle set Θ,

$$g\colon R \to \Theta \quad (9)$$

this mapping is clearly not a bijection. We will show this below.

First, the radius set R satisfies Equation (2). On the other hand, assuming Equation (5), the angle set Θ

$$g\colon r \to \theta \quad (10)$$

is clearly a bijection. However, when the arc length set A is fixed by the empty set $\emptyset_A$, from Equation (7), the angle set Θ is also an empty set $\emptyset_\Theta$; therefore, if we assume that

$$g\colon \emptyset_R \to \emptyset_\Theta \quad (11)$$

holds, on the other hand, the inverse mapping ($g - 1$) of mapping g,

$$g - 1\colon \emptyset_\Theta \to \emptyset_R \quad (12)$$

generally does not hold. In other words, Equation (11) is an injection. Therefore, Equation (9) is not a bijection.

Here, it is assumed that Equation (11) holds. Thus, let us show that Equation (11) does hold. If a straight line that makes the radius set R is on the baseline B, the reference point O is shared. Thus, based on the definition, $\Theta = \emptyset_\Theta = 0$ is clear. Here, if the radius R is 0, the radius set is clearly $R = \emptyset_R = 0$. Here, empty set $\emptyset = 0$ is handled by the standard definition of a general set.

Even if the radius set R is 0; i.e., the empty set $\emptyset_R$, the radius set R shares the reference point O on baseline L. At this time, clearly the arc length set A becomes the empty set $\emptyset_A$. In other words, for mapping h,

$$h\colon \emptyset_R \to \emptyset_A \quad (13)$$

holds, but

$$h^{-1}\colon \emptyset_A \to \emptyset_R \quad (14)$$

does not. On the other hand, from the relationship of the injectivity of Equation (13) and the bijectivity of Equation (8),

$$\emptyset_R \to \emptyset_A \to \emptyset_\Theta \quad (15)$$

holds. Therefore, the injectivity of Equation (11) holds.

Now, based on the definition, the angle set Θ is stipulated as

$$\Theta = \frac{A}{R} \qquad (16)$$

based on the ratio of the radius set R and the arc length set A; therefore, if we apply Equation (15) to Equation (16),

$$\Theta = \frac{A}{R} = \frac{0}{0} = 0 \qquad (17)$$

is obtained. As such, Theorem 1 is proven.

If radius R is constant, and both sides of Equation (16) are differentiated by time t, this can be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}t}\Theta = \frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}t}A \qquad (18)$$

Here, the left side is the angular velocity ω, while the differential form on the right side is the tangential velocity v. Thus, if we substitute these expressions, the above equation becomes

$$\omega = \frac{v}{R} \qquad (19)$$

Equation (19) expresses the relationship between the radius R of a rotating body, tangential velocity v, and angular velocity ω, but the relationship in Equation (17) is clearly angular velocity $\omega = 0$ when $R = 0$, showing that the center of a rotating body does not rotate. As such, Theorem 2 is proven.